DATA ASSOCIATION ALGORITHM FOR MULTISENSOR MULTITARGET TRACKING

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1. Introduction. In this paper an algorithm is presented for solving data association problem in multisensor multitarget tracking that follows a central level approach. Three non-collocated sensors are assumed, each of which sends at every scan a set of measurements to the central processor. The problem to be solved is to find out such a measurements partitioning which minimizes e negative log-likelihood cost function[1]. This function is a probabilistic estimation of the fact that associated measurements originate from one and the same target. The presented algorithm is an extension of the algorithm **A3** from [2] for dealing in the most complicated case with $P_D < 1$ (P_D **-** probability to detect an existing target) and $P_{FA} \neq 0$ (P_{FA} - probability for false alarm).

2. Problem formulation. We denote the sets of measurements received from each sensor by $\{Z^1\} \equiv \{Z_1^1, Z_2^1, ..., Z_{n_1}^1\}$, $\{Z^2\} \equiv \{Z_1^2, Z_2^2, ..., Z_{n_2}^2\}$, $\{Z^3\} \equiv \{Z_1^3, Z_2^3, ..., Z_{n_3}^3\}$. With no loss of generality it is assumed that $n_1 \geq n_2 \geq n_3$. Taking into account χ^2 -distribution of the weighted distances between any two measurements from different sets, we will decide that two measurements originate from one and the same target if the weighted distance between them is less then some appropriate chosen threshold Q. When we choose threshold Q we have to keep in mind that every value of Q is connected with some probability to lose a part of actual pairings.

In order to simplify the notations we add a zero-index in the cost matrix of the task. With this index we will denote these cases when some measurements from $\{Z^1\}$ are not associated with any of the measurements of $\{Z^2\}$ and $\{Z^3\}$ and vice versa. For example, if a term $A_{i_1 00}$ is included in obtained solution this implies that the measurement $Z_{i_1}^1$ is not associated with any of the measurements of $\{Z^2\}$ and/or $\{Z^3\}$.

Besides:

$$A_{i_1 00} = A_{0 i_2 0} = A_{00 i_3} = 0; \qquad A_{i_1 i_2 0} = 2\left(V_{i_1 i_2}^T R^{-1} V_{i_1 i_2}\right);$$

(1)
$$A_{i_1 0 i_3} = 2\left(V_{i_1 i_3}^T R^{-1} V_{i_1 i_3}\right); \qquad A_{0 i_2 i_3} = 2\left(V_{i_2 i_3}^T R^{-1} V_{i_2 i_3}\right);$$

$$A_{(i_1 i_2) i_3} = 2\left(V_{i_1 i_3}^T R^{-1} V_{i_1 i_3}\right) + 2\left(V_{i_2 i_3}^T R^{-1} V_{i_2 i_3}\right), \quad \text{for } i_1, i_2, i_3 \neq 0.$$

Here V is a vector connecting two measurements: $V_{i_1 i_2} = \left(Z_{i_1}^1 - Z_{i_2}^2 \right)$.

3. Proposed algorithm. This algorithm involves four main steps.

Step1: Minimize

$$\sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} A_{i_1 i_2 0} \delta_{i_1 i_2 0} \quad \text{subject to}$$

(2)
$$\sum_{i_1=1}^{n_1} \delta_{i_1 i_2 0} = 1 \quad \text{for every} \quad i_2 = 1 \div n \,;$$

$$\sum_{i_2=1}^{n_2} \delta_{i_1 i_2 0} = 1 \quad \text{for every} \quad i_1 = 1 \div n \,.$$

Here $\delta_{i_1 i_2 i_3}$ are binary variables taking values of 0 or 1. We can solve this problem using an extension of Munkres' algorithm [3]. Let us denote the obtained solution:

(3)
$$\Phi_1 = \sum_{i_1=0}^{n_1} A_{i_1 \beta_{i_1} 0} \,.$$

Here β_{i_1} is the measurement from the set $\left\{ Z^2 \right\}$ to which measurement i_1 from $\left\{ Z^1 \right\}$ is assigned.

Step2: Now we have to scan obtained solution and check every term in the sum of (3) for feasibility. Let us denote with $\{l\}$ the subset of $\{i_1\}$ for which

(4)
$$A_{l \beta_l 0} > Q \,,$$

where Q is an appropriate chosen threshold. This inequality corresponds to the unfeasible pairings and we will split every term from the sum of (3) which subjects to (4), i.e. the term $A_{l \beta_l 0}$ is split to $A_{l 0 0}$ and $A_{0 \beta_l 0}$.

Here, according to (1), $A_{l 0 0} = A_{0 \beta_l 0} = 0$. Now in the set $\left\{ i_1 \beta_{i_1} \right\}_{i_1 \neq 0}$ will remain only such terms for which $A_{l \beta_l 0} \leq Q$. Let us construct the following compound set $\{L\}$ whose elements are pairs of indices $\{L\} \equiv \left\{ i_1 \beta_{i_2} \right\}_{i_1 \neq 0} \cup \{l 0\} \cup \{0 \beta_l\}$. Then the final form of (3) after step2 will be

$$\Phi_1 = \sum_{(i_1 i_2) \in L} \sum_{i_3=0}^{n_3} A_{(i_1 i_2) i_3} \delta_{(i_1 i_2) i_3} \,.$$

Step3: Minimize

$$\sum_{(i_1 i_2) \in L} \sum_{i_3=0}^{n_3} A_{(i_1 i_2) i_3} \delta_{(i_1 i_2) i_3} \quad \text{subject to}$$

(5)
$$\sum_{(i_1 i_2) \in L} \delta_{(i_1 i_2) i_3} = 1 \quad \text{for every} \quad i_3 = 1 \div n_3$$

$$\sum_{i_3=0}^{n_3} \delta_{(i_1 i_2) i_3} = 1 \quad \text{for every} \quad (i_1 i_2) \in L \,.$$

Solving this assignment problem using once more the algorithm from [3] we obtain the final result

$$(6) \qquad \Phi^* = \sum_{(i_1 i_2) \in L} A_{(i_1 i_2)\beta_L} .$$

Step 4: At last we scan once more the obtained solution (6) and check it for feasibility but now we simply exclude from (6) the terms subject to (4). The remaining 3-tuples of indices we denote as a set D. So, the final form of the problem solution will be $\Phi^* = \sum_{(i_1 i_2 i_3) \in D} A_{(i_1 i_2 i_3)}$.

4. Improving the obtained solution. The solutions (3) and (6) contain probably unfeasible pairings. These results can be considerably improved by the simple beforehand treatment of the cost matrix. An illustration of this idea is given in Fig.1. In Fig.1, a a fragment of the cost matrix is shown. Les us assume the value of $Q = 9.21$ (i.e. in the case of two degrees of freedom the probability that any weighted distance, corresponding to correct pairing, will exceed this value, is 1%).

$$
\begin{array}{cccc}
m & n & \qquad & m \qquad n \\
\vdots & \vdots & & \vdots \qquad \vdots \\
k \ldots 13.2 \ldots 6.3 \ldots & & k \ldots 10 \ldots 6.3 \ldots \\
\vdots & \vdots & & \vdots \qquad \vdots \\
l \ldots 35.7 \ldots 11.8 \ldots & & l \ldots 10 \ldots 10 \ldots \\
\vdots & \vdots & & \vdots \qquad \vdots \\
(a) & & & (b)
\end{array}
$$

Fig.1

There are two alternative terms of this fragment to be included in the solution (Fig.1,a) : $A_{km} + A_{ln}$ or $A_{kn} + A_{lm}$. Besides, $A_{km} + A_{ln} = 25$ and $A_{kn} + A_{lm} = 42$. In spite of the fact that A_{km} and A_{ln} are both greater then Q , the algorithm will include them in the solution (the first alternative), because they add to the total sum less quantity. This coincide with the objective criterion, used here: "the minimal sum of A-terms". But our actual aim is to find out "as many as possible correct association" Moreover, every association which is subject to relation (4) is equally inadmissible, no matter what is the quantity of corresponding A-term. Now, let us replace every term of the fragment of the cost matrix (Fig.1, a), which is subject to equation (4), with one and the same value Q^*, rather than Q, say $Q^* = 10$ (Fig.1,b). In this case, from the two possible alternatives the algorithm will choose the second one: $A_{kn} + A_{lm}$. Thus, the criterion "minimal sum of A-terms" fully coincides with our actual aim "to out as many as possible correct associations".

5. Simulation results. Averaging over more than 1000 runs of Monte Carlo simulation program we have obtained numerical results proving reliability of the proposed algorithm. Another run of experiments have been carried out in order to prove improvement of the results by means of cost matrix treatment (Fig.2). Two graphics represent 36 and 64 targets number cases respectively. All the tests were performed with $P_D = 0.7$ .Any of the
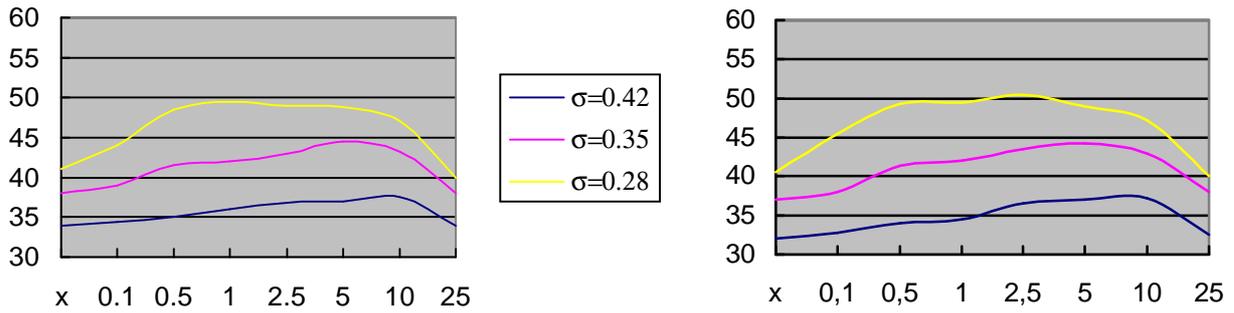
Fig. 2

curves is connected with particular value of the standard deviation σ, normalized by the average distance between the targets.

On the vertical axis the probability (in percent) of correct association is plotted. The points on the horizontal axis correspond to decreasing values of the threshold Q and the numbers below these axis denote probability (in percent) of "losing" actual pairings, connected with the corresponding value of Q. The point marked with '×' denotes the case when no treatment of the cost matrix is used. As it can be seen, the improvement of the results depends on the values of Q. The values of Q corresponding to probability of 1% and 2.5% prove to be more preferable.

6. Conclusions. In this paper an algorithm for solving 3D assignment problem has been proposed. The algorithm is intended to work in dense target environment with missing detections and false alarms. A simple technique for matrix treatment has been developed which improves the algorithm performance. Numerical results exposed in the paper prove reliability of the algorithm and its high performance estimation in terms of correct association probability.

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