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SOME ALGORITHMS FOR SOLVING ASSIGNMENT PROBLEM IN MULTISENSOR MULTITARGET TRACKING

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Introduction. This paper concerns the problem of data association (DA) which is the central problem of multitarget tracking. Its simplest form can be stated as follows: to associate a list of measurements to a list of target positions by an unknown, random permutation [1-3]. This measurement-target association problem is formulated as one of maximizing the joint likelihood function of the measurements partition. Mathematically this formulation of data association problem leads to the well-known assignment problem. In this case, maximization of joint likelihood function is replaced by minimization of negative log-likelihood function [1-4]. However, the problem of associating data from multiple noncollocated sensors, where the target number and target positions are unknown and targets are closely spaced is not well studied.

Some special features of the problem under consideration arise in a case of dense target environment. Particular attention deserves the extent of target density when the optimal solution does not coincide with the actual measurements distribution.

Problem formulation. Hereafter we will confine the multisensor problem to three sensors with known positions. Three sensors in multitarget multisensor problem are, in a sense, an optimal number of sensors, as it is pointed out by HALL in [4]. We assume that number of targets is unknown. We consider a planar surveillance area, wherein each target position is described by its Cartesian coordinates, *x* and *y*. As a matter of fact, a common type of radars can measure the range and the azimuth. Using coordinate transformation, however, we can easily obtain Cartesian target coordinates. We assume no missing detection and false alarms in order to simplify algorithm's description.

We denote the set of measurements received from any sensor by $\{Z_{i_s}^{s}\}$, where s =1, 2 or 3; $n = 1,2,\ldots,n$ and n is the number of sensors' reports. The solving of the problem consists of finding such a partitioning of the measurements in 3-tuples $\{Z_{i_1}^{1}, Z_{i_2}^{2}, Z_{i_3}^{3}\}$ that each measurement originates from one and the same target. Assuming Gaussian statistics for measurements

$$f(v) = \frac{1}{2\pi\sqrt{|R|}} \exp\left(-\frac{1}{2} v^T R^{-1} v\right), \tag{1}$$

where v is a residual vector between the measurements and R is a residual covariance matrix. We can use as a measure of similarity of these two measurements the Mahalanobis distance $v^T R^{-1} v$ [4].

The minimization of this distance maximizes the joint likelihood function (1). The measure that a given 3-tuple of measurements - $Z^1_{i_1}, Z^2_{i_2}$ and $Z^3_{i_3}$, originates from the same target will be the sum $A_{i_1 i_2 i_3}$ of these Mahalanobis distances.

The stated above problem can be mathematically formulated as a following three-dimensional problem:

minimize

(2) $$\sum_{i_1=1}^{n}\sum_{i_2=1}^{n}\sum_{i_3=1}^{n} A_{i_1 i_2 i_3}\delta_{i_1 i_2 i_3} \qquad \text{subject to}$$

(3) $$\sum_{i_1=1}^{n}\sum_{i_2=1}^{n}\delta_{i_1 i_2 i_3} = 1; \qquad \sum_{i_1=1}^{n}\sum_{i_3=1}^{n}\delta_{i_1 i_2 i_3} = 1; \qquad \sum_{i_2=1}^{n}\sum_{i_3=1}^{n}\delta_{i_1 i_2 i_3} = 1;$$

$$\text{for all } i_1, i_2, i_3 = 1 \div n.$$

Here $\delta_{i_1 i_2 i_3}$ are binary variables, taking values 0 or 1.

Proposed algorithms. To our knowledge there are no algorithms for finding optimal solution to the problem stated above. What is more, it can be shown that 3D AP is NP-complete [5]. On the other hand, in a dens target case the optimum does not provide the actual measurements' distribution. We propose here heuristic strategy in order to create useful algorithms for finding suboptimal decisions. They have a good reliability and high performance estimation in terms of correct association (CA) probability.

ALGORITHM **A1.** This algorithm involves successive solving of two 2-dimensional problems:

***step 1* :** to minimize $\sum_{i_1=1}^{n}\sum_{i_2=1}^{n} A_{i_1 i_2 1}\delta_{i_1 i_2 1}$ subject to

$$\sum_{i_1=1}^{n}\delta_{i_1 i_2 1} = 1, \qquad \sum_{i_2=1}^{n}\delta_{i_1 i_2 1} = 1, \qquad \text{for all } i_1, i_2 = 1 \div \text{n}.$$

We can solve this problem using one of the well known assignment algorithms, e.g. an extension of Munkres' algorithm [6]. Let us denote the obtained solution by

(4) $$\Phi_1 = \sum A_{i_1 \beta_{i_1} 1}.$$

Here β_{i_1} is the measurement of sensor 2 to which measurement i_1 of sensor 1 is assigned.

***step 2* :** to minimize $\sum_{i_1=1}^{n}\sum_{i_3=1}^{n} A_{i_1 1 i_3}\delta_{i_1 1 i_3}$ subject to

$$\sum_{i_1=1}^{n}\delta_{i_1 1 i_3} = 1, \qquad \sum_{i_3=1}^{n}\delta_{i_1 1 i_3} = 1, \qquad \text{for all } i_1, i_3 = 1 \div n.$$

Using the same algorithm we solve the problem and denote its solution by

(5)
$$\Phi_2 = \sum_{i_1=1}^{n} A_{i_1 1 \gamma_{i_1}} .$$

Analogous to equation (4), γ_{i_1} is the measurement of sensor 3 to which measurement i_1 of sensor 1 is assigned.

step 3 **:** combining (4) and (5) we obtain suboptimal solution to a 3-dimensional problem

(6)
$$\Phi_I^* \equiv \{\Phi_1 \otimes \Phi_2\} = \sum_{\mathrm{i}_1=1}^{n} A_{i_1 \beta_{i_1} \gamma_{i_1}} .$$

We can improve the results obtained by solving another 2-dimensional problem.

ALGORITHM **A2.** In addition to the procedure described above (step1 and step2) we can solve the following problem:

step 3 **:** to minimize $\sum_{i_2=1}^{n} \sum_{i_3=1}^{n} A_{1 i_2 i_3} \delta_{1 i_2 i_3}$ subject to

$$\sum_{i_2=1}^{n} \delta_{1 i_2 i_3} = 1, \qquad \sum_{i_3=1}^{n} \delta_{1 i_2 i_3} = 1, \qquad \text{for all} \quad i_2, i_3 = 1 \div n.$$

Let us denote the obtained decision by

(7)
$$\Phi_3 = \sum_{\mathrm{i}_2=1}^{n} A_{1 i_2 \gamma_{i_2}} .$$

Now, in addition to (6), we can construct two more 3-dimentional solutions and to choose the best one

$$\Phi_{II}^* \equiv \{\Phi_1 \otimes \Phi_3\} = \sum_{\mathrm{i}_2=1}^{n} A_{\alpha_{i_2} i_2 \gamma_{i_2}} \; ; \qquad \Phi_{III}^* \equiv \{\Phi_2 \otimes \Phi_3\} = \sum_{\mathrm{i}_3=1}^{n} A_{\alpha_{i_3} \beta_{i_3} i_3} .$$

Here a question arises whether the solutions Φ_I^*, Φ_{II}^* and Φ_{III}^* are feasible, i.e. whether they satisfy the constraints (3). Let us examine solution (6) with respect to the first term of constrained (3). Indices in (6) mean that for each $\gamma_{i_1} \in \{1,2,...,\mathrm{n}\}$ in the "plane" (i_1, i_2) there is no more than one "participant" $A_{i_1 \beta_{i_1} \gamma_{i_1}}$ in the sum of (6),which is in accordance with the constraints. Analogous reasoning can be done another two terms of (3).

In the algorithms described above every 2-dimensional solution is derived independently from the others. The next step of improving the assignment technique is to solve the second 2-dimensional problem in **A1**, using results obtained from the first solution.

ALGORITHM **A3**. Here we start with ***step 1*** of **A1**. But in ***step 2*** we solve the following assignment problem:

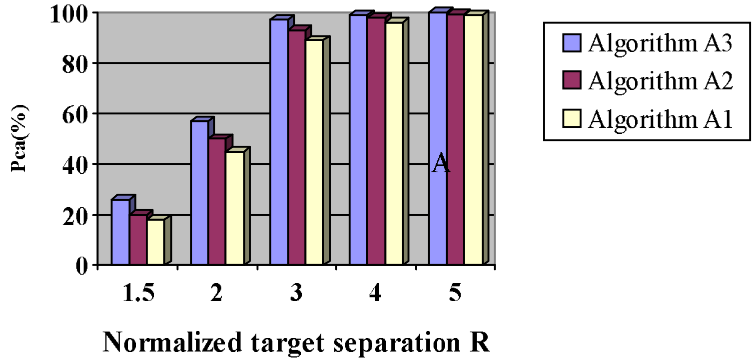

Fig.1 Fraction of correct association versus the ratio R of target separation to measurement noise standard deviation for 36 targets

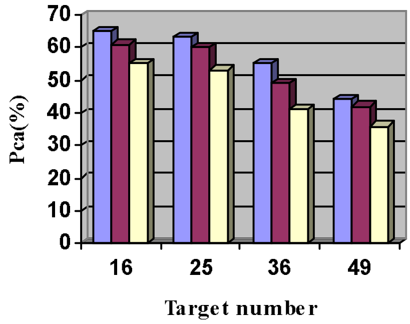

Fig.2 Fraction of correct association versus different number of targets for $R = 2$

***step 2*:** minimize $\sum_{i_1=1}^{n}\sum_{i_3=1}^{n} A_{i_1\beta_{i_1}i_3}\delta_{i_1\beta_{i_1}i_3}$, subject to

$$\sum_{i_1=1}^{n}\delta_{i_1\beta_{i_1}i_3} = 1, \qquad \sum_{i_3=1}^{n}\delta_{i_1\beta_{i_1}i_3} = 1, \qquad \text{for all} \quad i_1, i_3 = 1 \div n.$$

As a result we obtain a solution of 3-dimentional case directly $\Phi^* = \sum_{i_1=1}^{n} A_{i_1\beta_{i_1}\gamma_{i_1}}$.

Monte Carlo verification. The numerical examples we have chosen contain different number of targets but with common scenario frame. In that scenario the targets are disposed in the nodes of rectangular net. Two figures illustrate relative performance of the three algorithms on randomly generated sensor reports. Each point in the figures was obtained by averaging over 100 independent programme runs.

Conclusions. In this paper computationally simple algorithms for solving 3-dimentional assignment problem have been proposed. Averaging more than 1000 programme runs we obtained experimentally a bound of computational time less than $O(n^3)$. For dealing with a more complicated case including missing detections and false alarms some extensions of described algorithms have to be made.

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