Fusion Formula for DSN Estimates and Tracking[1]

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Abstract. In distributed sensor networks (DSN), a set of nodes implements two roles: a) to collect and process data from local sensors and b) to communicate with other nodes. When communicating data is received at a given node this data have to be fused with the local processed data for achieving improved state estimate for tracking targets. In this paper an attempt is made for derivation of fusion formula in its full extent for fusing information from arbitrary number of nodes.
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1. Introduction. More then a decade, there has been growing interest in distributed estimation problems. The traditional approach in estimation using information from multiple sensors has been centralized [1]. The measurements generated by all sensors is assumed to be sent to a central computer where processing is carried out. In contrast, in distributed sensor network (DSN) a set of local processors perform tracking functions using the local sensor data. In addition, each local processor (or estimation node) communicates the processing results to other estimation nodes according to some communication strategy [1], [2], [3] and to some network architecture [5]. The centralized approach is known to be optimal [2],[4],[5] since central processor gather and processes all raw measurements. On the other hand, DSN approach has some advantages over centralized approach: a) it is reliable since there is not a single central site whose failure may doom the entire system; and b) communication is cheaper since the processed results, and not raw measurements, are communicated. This processed results are, as a matter of fact, information state, i.e. state distributions and probabilities. And an appropriate objective for fusion in DSN case is to reconstruct the optimal information state based on the information states received from the other nodes. In this paper we propose derivation of more suitable form of the fusion formula for arbitrary number of information processing nodes in comparison with [3] and [4]

The structure of the paper is as follows. In Section 2 we describe the information fusion problem. In Section 3 we derive the fusion formula for arbitrary number of nodes to be fused. In the last Section 4 we propose a discussion.

2. Problem formulation. We state the information fusion problem faced by each *information processing node*. Let $N = \{1,2,\ldots,n\}$ be a finite set of such processing nodes and S be a finite set of sensors (or *information sources*). Each processing node receives measurements from a set of *local sensors* and never share any sensor of this set with another processing node, i.e. if m and n are two processing nodes and S_m and S_n are their own local sensors sets, our assumption is

$$S_m \cap S_n = \varnothing .$$

We assume that node i receives a measurement $z(t,i)$ from one of its local sensors at time $t \in T_{0i}$ and at time $t \in T_{ji}$ receives information from node j. The set

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T_{0i} is observation time set and the set T_{ji} is reception time set. Additional notation is the set T_i of time instants at which the information of node i changes, i.e.

$$T_i = \bigcup_{j=0}^{m_i} T_{ji}$$

and more common set

$$T = \bigcup_{i=1}^{N} T_i$$

of time instants when changes of information occur for some node in the system.

Now two important assumptions follows concerning measurements and communication processes.

Concerning measurements. The measurements are conditionally independent given the estimated process $x(.)$. Particularly, to denote with J the set of all measurement indices, i.e.

$$J = \{(t,i) | t \in T_{0i}, i \in N\}.$$

Then for any arbitrary subset A of J the random measurement vector

$$Z(A) \equiv (z(t,i),(t,i) \in A) \quad \text{satisfies}$$

$$p(Z(A)|x(t), t \in T) = \prod_{(t,i) \in A} p(z(t,i)|x(t)).$$

This assumption helps as to completely characterized the observation process by the conditional probabilities (or probability densities) $p(z(t,i)|x(t))$.

Concerning communication. We assume that the information transmitted between the nodes is conditional probabilities (or the sufficient statistics) of the process $x(.)$. It can be shown [1] that the probabilities are conditioned on the cumulative local measurements plus the measurements from the nodes from whom messages have been received.

To include some additional notations. The couple (t,i) defines the moment when information for node i changes. The change may occur because of measurements reception or of information transmitted from another node. To define

$$K_i = \{(t,i), t \in T_i\} \text{ for } i = 1,\ldots,N \text{, and}$$

$$K = \bigcup_{i=1}^{n} K_i .$$

The K can be accepted as a total index set of all important events in DSN. For each index (t,i) in K, a subset $J(t,i)$ of J is the *cumulative measurement index set for node i at time t* if all the measurements indices in $J(t,i)$ are available to node i by direct observation or via transmitted information. The sets $J(t,i)$ satisfies some properties as follows:

a) For fixed i, if $s < t$, then $J(s,i) \subseteq J(t,i)$.

b) If $J(s,i) \cap J(t,i) \neq \varnothing$, this specifies common information because of communication in the past, but this common information at node i may not be a cumulative measurement index set $J(r,k)$ for any (r,k) in K.

c) Suppose $t > r$, $t > s$ and t is a transmission time from j to i, than $J(r,i) \cup J(s,j) \subseteq J(t,i)$.

Finally, the actual measurement information to the node i at time t is represented by the *cumulative measurement vector*

$$Z(t,i) = \{z(s,j),(s,j) \in J(t,i)\}.$$

3. Formula derivation. In this section we propose derivation of the fusion formula for arbitrary number of measurement vectors Z_i. and for static estimation problems. Our goal can be summarized in the next way: to compute $p(y(t,i)|Z(t,i))$ as new measurements arrive or when messages are received from other nodes. When a sensor measurements is obtained, updating is straightforward using Bayes' rule. When, however, the node i receives the conditional probability $p(y|Z(s,j))$ from node j, updating is more complicated. In particular, $p(y|Z(s,j))$ may contain information previously originating from node i. To compute, for example, $p(y|Z(t,i)\cup Z(s,j))$ it is necessary to remove the redundancy in the two probabilities.

Here we assume the random process is static, i.e. $x(t)=x$ for all t. In [1] the fusion formula is derived for two measurement subvectors Z_1 and Z_2 defined on given measurement index sets J_1 and J_2 (Lemma 1):

$$p(x|Z_1 \cup Z_2) = C\,\frac{p(x|Z_1)p(x|Z_2)}{p(x|Z_1 \cap Z_2)} \tag{1}$$

Replacing in the upper equation Z_2 with $Z_2 \cup Z_3$ the fusion formula for three subvectors can be derived (Lemma 3 in [1]).

Now, we start with the proof of the next form of the fusion formula for arbitrary number of measurement subvectors Z_i:

Suppose $Z_1, Z_2, \ldots, Z_N$ are measurement subvectors defined on given cumulative measurement index sets $J_1, J_2, \ldots, J_N$ of Z. Then

$$p\left(x\middle|\bigcup_{i=1}^{N} Z_i\right) = C\prod_{k=1}^{N}\left[\prod_{i_1\ldots i_k}^{\binom{N}{k}} p\left(x\middle|Z_{i_1}\cap Z_{i_2}\cap\ldots\cap Z_{i_k}\right)\right]^{(-1)^{k-1}}, \tag{2}$$

where

$$C = \frac{1}{p\left(\bigcup_{i=1}^{N} Z_i\right)}\prod_{k=1}^{N}\left[\prod_{i_1\ldots i_k}^{\binom{N}{k}} p\left(Z_{i_1}\cap Z_{i_2}\cap\ldots\cap Z_{i_k}\right)\right]^{(-1)^{k-1}}.$$

Notation $\prod_{i_1\ldots i_k}^{\binom{N}{k}}$ means multiplication of all possible combination of k subvectors out of N in conditional probabilities p.

Proof

Firs, following the proof of Lemma 1 in [1] we can express normalization constant C as:

$$C = \frac{1}{p(Z_1 \cup Z_2)}\cdot\frac{p(Z_1)p(Z_2)}{p(Z_1 \cap Z_2)} \tag{3}$$

Replacing C from (3) in (1) and rearranging the multipliers we obtain the next form of fusion formula for two measurement subvectors:

$$p(x|Z_1 \cup Z_2) = \frac{1}{p(Z_1 \cup Z_2)}\,\frac{p(x|Z_1)p(Z_1)\,p(x|Z_2)p(Z_2)}{p(x|Z_1 \cap Z_2)p(Z_1 \cap Z_2)}, \qquad \text{or}$$

$$p\left(x\middle|Z_1 \cup Z_2\right)p\left(Z_1 \cup Z_2\right) = \frac{p\left(x\middle|Z_1\right)p\left(Z_1\right)\, p\left(x\middle|Z_2\right)p\left(Z_2\right)}{p\left(x\middle|Z_1 \cap Z_2\right)p\left(Z_1 \cap Z_2\right)} \tag{4}$$

Now, reminding that $p(A|B)p(B) = p(B|A)p(B)$ is probability of events A and B to occur simultaneously to accept for convenience the next notation

$$p(A|B)p(B) = p(B|A)p(B) = p(A;B).$$

Using this notation to rewrite the equation (1)

$$p\left(x;Z_1 \cup Z_2\right) = \frac{p\left(x;Z_1\right)p\left(x;Z_2\right)}{p\left(x;Z_1 \cap Z_2\right)} \tag{5}$$

Here, as in Lemma 3 in [1] replacing Z_2 with $Z_2 \cup Z_3$ we obtain the fusion formula for three subvectors:

$$p\left(x;Z_1 \cup Z_2 \cup Z_3\right) = \frac{p\left(x;Z_1\right)p\left(x;Z_2\right)p\left(Z_3\right)p\left(x;Z_1 \cap Z_2 \cap Z_3\right)}{p\left(x;Z_1 \cap Z_2\right)p\left(x;Z_1 \cap Z_3\right)p\left(x;Z_2 \cap Z_3\right)} \tag{6}$$

For the multiplier in the numerator $p\left(x;Z_2\right) \to p\left(x;Z_2 \cup Z_3\right)$ we apply the equation (5) directly. After replacement the denominator will be transform as

$$p\left(x;Z_1 \cap Z_2\right) \to p\left(x;Z_1 \cap \left(Z_2 \cup Z_3\right)\right) = p\left(x;\left(Z_1 \cap Z_2\right) \cup \left(Z_1 \cap Z_3\right)\right).$$

For the two sets in the parentheses in above equation we apply once more equation (5) and calling to mind that $\left(Z_1 \cap Z_2\right) \cap \left(Z_1 \cap Z_3\right) = Z_1 \cap Z_2 \cap Z_3$ we obtain (6).

Following the mathematical induction approach we assume that formula (2) is true for N measurement subvectors and will proof that it is true for $N+1$ subvectors. First to rewrite (2) according to accepting notation as for two subvectors case

$$p\left(x;\bigcup_{i=1}^{N} Z_i\right) = \prod_{k=1}^{N}\left[\prod_{i_1 \ldots i_k}^{\binom{N}{k}} p\left(x;Z_{i_1} \cap Z_{i_2} \cap \ldots \cap Z_{i_k}\right)\right]^{(-1)^{k-1}} . \tag{7}$$

We will proceed as in three subvectors case replacing Z_N with $Z_N \cup Z_{N+1}$. To analyze increasing of the number of multipliers for every particular k. For a given k $\binom{N}{k}$ multipliers are generated. The subvector Z_N appears in $\binom{N-1}{k-1}$ out of $\binom{N}{k}$. For arbitrary multiplier the mentioned above replacement will give

$$\left(Z_{i_1} \cap \ldots Z_{i_{k-1}}\left(Z_N \cup Z_{N+1}\right)\right) = \left(Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_N\right) \cup \left(Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_{N+1}\right) \tag{8}$$

Applying equation (5) for probabilities corresponding to (8) we obtain

$$\begin{aligned} &p\left(x;\left(Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_N\right) \cup \left(Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_{N+1}\right)\right) = \\ &= \frac{p\left(x;Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_N\right)p\left(x;Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_{N+1}\right)}{p\left(x;Z_{i_1} \cap \ldots Z_{i_{k-1}} \cap Z_N \cap Z_{N+1}\right)} \end{aligned} \tag{9}$$

The conclusion is straightforward: for a given k any of the $\binom{N-1}{k-1}$ multipliers is split into two multipliers for k-subvectors case and in addition one multiplier is created for (k+1)-subvectors case. To express this result for two consecutive values of index - *k-1* and *k*

$$\binom{N}{k-1} : \leftarrow + \binom{N-1}{k-2} \underset{\downarrow}{+} \binom{N-1}{k-2}$$
$$\binom{N}{k} : \leftarrow + \binom{N-1}{k-1} \underset{\downarrow}{+} \binom{N-1}{k-1}$$

The horizontal arrow means that the second term remains in the same row and the vertical arrow means that the third term is added to the next row. It is clear that the number of multipliers for the given k will increase with

$$\binom{N-1}{k-1} + \binom{N-1}{k-2}.$$

Adding this increment to the $\binom{N}{k}$ will give

$$\binom{N}{k} + \binom{N-1}{k-1} + \binom{N-1}{k-2} = \binom{N+1}{k} \tag{10}$$

Now we can replace in (7) $\binom{N}{k}$ with $\binom{N+1}{k}$ in the multiplication symbol in the brackets. To consider the case $k=N$. Their is one term in this group which is split into two terms for case $k=N$ and one additional term for the new case $k=N+1$, which correspond to replacing N with $N+1$ in the multiplication symbol in front of brackets. Following this notations we receive the main result in this paper

$$p\left(x; \bigcup_{i=1}^{N+1} Z_i \right) = \prod_{k=1}^{N+1} \left[\prod_{i_1 \dots i_k}^{\binom{N+1}{k}} p\left(x; Z_{i_1} \cap Z_{i_2} \cap \ldots \cap Z_{i_k} \right) \right]^{(-1)^{k-1}} \tag{11}$$

And stepping back from the notations in deriving (5) we can rewrite the fusion formula for arbitrary number of measurements vectors containing conditional probabilities and normalization constant

$$p\left(x \middle| \bigcup_{i=1}^{N+1} Z_i \right) = C \prod_{k=1}^{N+1} \left[\prod_{i_1 \dots i_k}^{\binom{N+1}{k}} p\left(x \middle| Z_{i_1} \cap Z_{i_2} \cap \ldots \cap Z_{i_k} \right) \right]^{(-1)^{k-1}} \tag{12}$$

where

$$C = \frac{1}{p\left(\bigcup_{i=1}^{N+1} Z_i \right)} \prod_{k=1}^{N+1} \left[\prod_{i_1 \dots i_k}^{\binom{N+1}{k}} p\left(Z_{i_1} \cap Z_{i_2} \cap \ldots \cap Z_{i_k} \right) \right]^{(-1)^{k-1}} .$$

which proves the formula (2).

4.Discussion. The fusion formula for arbitrary number of information processing nodes have been proposed (without precise derivation) in [3] and [4]

$$p\left(x\middle|\bigcup_{i=1}^{n} Z_i\right)=C^{-1}\prod_{i=1}^{n}\left(\prod_{N\in N_i^n} p\left(x\middle|\bigcap_{j\in N} Z_j\right)\right)^{(-1)^{i-1}} \tag{13}$$

or another form

$$p\left(x\middle|\bigcup_{i\in I} Z^{(i)}\right)=c\prod_{i\in \bar{I}} p\left(x\middle|Z^{(i)}\right)^{\alpha(i)} \tag{14}$$

where I is the set of all immediate predecessors nods and $\bar{I}$ a subset of I of these predecessors which send information to the consider node.

In the process of computation of this formula the mentioned above subsets of course have to be determined. This task, however, is naturally to be a part of an algorithm for computation of considered formula. Starting from equation (12) such an algorithm has to recover the valid intersections and next to discover to which nodes this intersections correspond. On the base of equation (12) we developed a compact recursive algorithm for computing the fused estimation. In this algorithm is assumed that there exist some routine for recovering whether given intersection is empty or not. In the next stages we include some steps of algorithm presented in [4]. This algorithm will appear in the following (companion) paper.

5.Conclusions. A simple and rigorous derivation of fusion formula for arbitrary number of measurement vectors has presented in this paper. Derived formula is a base for constructing a compact recursive algorithm for its computation. In the mentioned formula a static case is considered. this formulation is starting point for considering more common and more complicated case - dynamic estimation problems in DSN. For example, if we ignore the process noise and with beforehand extrapolation of estimates to a common time the presented derivation can be directly used. And with some additional assumptions a most common formulation for dynamic estimation problems in DSN can be achieved.

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